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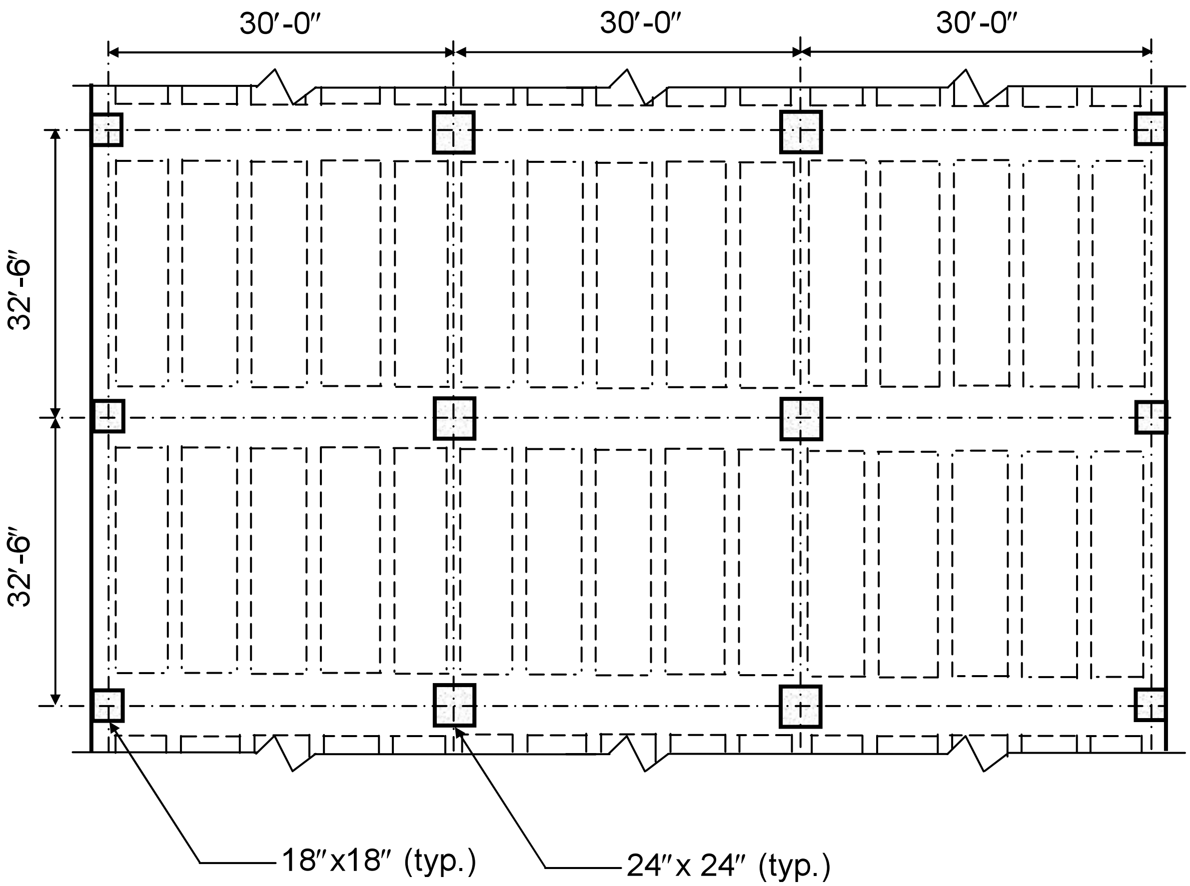
Beams and One-Way Slabs

The following example illustrates the design methods presented in the PCA book "Simplified Design - Reinforced Concrete Buildings of Moderate Size and Height" third edition. Unless otherwise noted, all referenced table, figure, and equation numbers are from that book.

Example Building

Below is a partial plan of a typical floor in a cast-in-place reinforced concrete building. The floor framing consists of wide-module joists and beams. In this example, the beams are designed and detailed for the combined effects of gravity and lateral (wind) loads according to ACI 318-05.



Design Data

Materials

- Concrete: normal weight (150 pcf), $^3/_4$ - in. maximum aggregate, f'_c = 4,000 psi
- Mild reinforcing steel: Grade 60 (f_y = 60,000 psi)

Loads

- Joists (16 + $4^1/_2$ x 6 + 66) = 76.6 psf
- Superimposed dead loads = 30 psf
- Live load = 100 psf
- Wind loads: per ASCE 7-02

Gravity Load Analysis

The coefficients of ACI Sect. 8.3 are utilized to compute the bending moments and shear forces along the length of the beam. From preliminary calculations, the beams are assumed to be 36 x 20.5 in. Live load reduction is taken per ASCE 7-02.

$$\text{Beam weight} = \frac{\frac{36 \times 20.5}{144} \times 150}{32.5} = 23.7 \text{ psf}$$

Live load reduction per ASCE 7-02 Sect. 4.8:

$$L = L_O \left(0.25 + \frac{15}{\sqrt{K_{LL} A_T}}\right)$$

From Figure C4 of ASCE 7-02, K_{LL} = live load element factor = 2 for interior beams

A_T = tributary area = 32.5 x 30 = 975 ft^2

$K_{LL}A_T$ = 2 x 975 = 1,950 ft^2 > 400 ft^2

$$L = L_O \left(0.25 + \frac{15}{\sqrt{1{,}950}}\right) = 0.59 L_O$$

Since the beams support only one floor, L shall not be less than $0.50L_O$.

Therefore, L = 0.59 x 100 = 59 psf.

Total factored load w_u:

w_u = 1.2(76.6 + 23.7 + 30) + 1.6(59)

= 250.8 psf

= 250.8 x 32.5/1,000 = 8.15 klf

Factored reactions per ACI Sect. 8.3 (see Figs. 2-3 through 2-7):

Neg. M_u at ext. support = $w_u \ell_n^2/16$

= 8.15 x 28.25^2/16

= 406.5 ft-kips

Pos. M_u at end span = $w_u \ell_n^2/14$

= 8.15 x 28.25^2/14

= 464.6 ft-kips

Neg. M_u at first int. col. $= w_u \ell_n^2/10^*$

$= 8.15 \times 28.125^2/10$

$= 644.7$ ft-kips

*Average of adjacent clear spans

Pos. M_u at int. span $= w_u \ell_n^2/16$

$= 8.15 \times 28^2/16$

$= 399.4$ ft-kips

V_u at exterior col. $= w_u \ell_n/2$

$= 8.15 \times 28.25/2$

$= 115.1$ kips

V_u at first interior col. $= 1.15 w_u \ell_n/2$

$= 1.15 \times 115.1$

$= 132.4$ kips

Wind Load Analysis

As noted above, wind forces are computed per ASCE 7-02. Calculations yield the following reactions:

$M_w = \pm\ 90.3$ ft-kips

$V_w = 6.0$ kips

Design for Flexure

Sizing the cross-section

Per ACI Table 9.5(a), minimum thickness = $\ell/18.5 = (30 \times 12)/18.5 = 19.5$ in.

Since joists are 20.5 in. deep, use 20.5-in. depth for the beams for formwork economy.

With d = 20.5 – 2.5 = 18 in., solving for b using maximum M_u along span (note: gravity moment combination governs):

$bd^2 = 20M_u$

$b = 20 \times 644.7/18^2 = 39.8$ in. > 36 in.

This implies that using a 36-in. wide beam, ρ will be greater than $0.5\rho_{max}$.

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Check minimum width based on $\rho = \rho_{max}$ (see Chapter 3 of the PCA publication *Simplified Design of Reinforced Concrete Buildings of Moderate Size and Height* for derivation):

$bd^2 = 14.6M_u$

$b = 14.6 \times 644.7/18^2 = 29.1$ in. < 36 in.

This implies that ρ will be less than ρ_{max}.

Use 36 x 20.5 in. beam.

Required Reinforcement

Beam moments along the span are summarized in the table below.

No.	Load Case	Location	End Span (ft-kips)	Interior span (ft-kips)
	Dead (D)	Exterior negative	-211.2	—
		Positive	241.4	207.5
		Interior negative	-335.0	-304.9
	Live (L)	Exterior negative	-95.6	—
		Positive	109.3	94.0
		Interior negative	-151.7	-138.1
	Wind (W)	Exterior negative	±90.3	—
		Positive	—	—
		Interior negative	±90.3	±90.3
No.	Load Combination			
1	1.2D + 1.6L (9-2)	Exterior negative	-406.4	—
		Positive	464.6	399.4
		Interior negative	-644.7	-586.8
2	1.2D + 0.5L + 1.6W (9-4)	Exterior negative	-156.8 -445.7	—
		Positive	344.3	296.0
		Interior negative	-622.3 -333.4	-290.5 -579.4
3	0.9D + 1.6W (9-6)	Exterior negative	-45.6 -334.6	—
		Positive	217.3	186.8
		Interior negative	-446 -157	-129.4 -418.9

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Required reinforcement, is summarized in the table below. Tables 3-2 and 3-3 are utilized to ensure that the number of bars chosen conforms to the code requirements for cover and spacing.

Location		M_u (ft-kips)	A_s (in.^{2})*	Reinforcement
End Span	Exterior negative	-445.7	6.19	8-No. 8
	Positive	464.6	6.45	9-No. 8
	Interior negative	-644.7	8.95	12-No. 8
Interior Span	Positive	399.4	5.54	7-No. 8

*$A_s = M_u/4d$

Min. $A_s = 3\sqrt{4{,}000} \times 36 \times 18/60{,}000 = 2.05$ in.^{2}

$= 200 \times 36 \times 18/60{,}000 = 2.16$ in.^{2} (governs)

Max. $A_s = 0.0206 \times 36 \times 18 = 13.35$ in.^{2}

For example, at the exterior negative location in the end span, the required $A_s = M_u/4d = 445.7/(4 \times 18) = 6.19$ in.^{2} Eight No. 8 bars provides 6.32 in.^{2}. From Table 3-2, the minimum number of No. 8 bars for a 36-in. wide beam is 5. Similarly, from Table 3-3, the maximum number of No. 8 bars is 16. Therefore, 8-No. 8 bars are adequate.

Design for Shear

Shear design is illustrated by determining the requirements at the exterior face of the interior column.

$V_u = 1.2D + 1.6L = 132.4$ kips (governs)

V_u at d from face $= 132.4 - 8.15(18/12)$

$= 120.2$ kips

Max. $(\phi V_c + \phi V_s) = \phi 10\sqrt{f'_c}\, b_w d = 307.4$ kips

$\phi V_c = \phi 2\sqrt{f'_c}\, b_w d = 61.5$ kips

Required $\phi V_s = 120.2 - 61.5 = 58.7$ kips

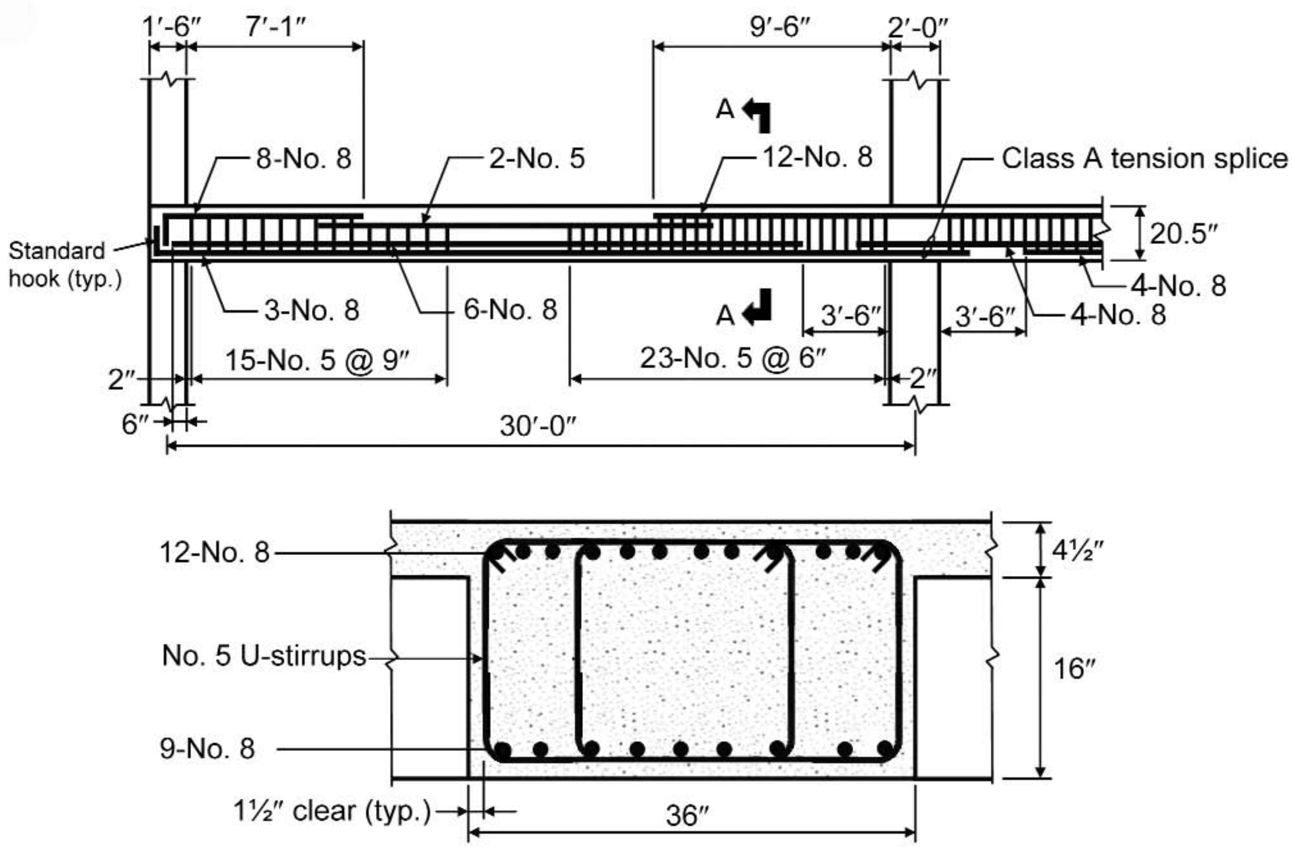
From Table 3-8, No. 5 U-stirrups at d/3 provides $\phi V_s = 84$ kips > 58.7 kips. Length over which stirrups are required = [120.2 – (61.5/2)]/8.15 = 11 ft from face of support.

Use No. 5 stirrups @ 6 in.

Reinforcement Details

The figure below shows the reinforcement details for the beam. The bar lengths are computed from Fig. 8-3 of the PCA publication *Simplified Design of Reinforced Concrete Buildings of Moderate Size and Height*. In lieu of computing the bar lengths in accordance with ACI Sects. 12.10 through 12.12, 2-No. 5 bars are provided within the center portion of the span to account for any variations in required bar lengths due to wind effects. For overall economy, it may be worthwhile to forego the No. 5 bars and determine the actual bar lengths per the above ACI sections.

Since the beams are part of the primary lateral-load-resisting system, ACI Sect. 12.11.2 requires that at least one-fourth of the positive moment reinforcement extend into the support and be anchored to develop f_y in tension at the face of the support.



Section A-A