Columns

Portland Cement Association

The following examples illustrate the design methods presented in the PCA book "Simplified Design - Reinforced Concrete Buildings of Moderate Size and Height" third edition. Unless otherwise noted, all referenced table, figure, and equation numbers are from that book. The examples presented here are for columns.

Examples for walls are available on our Web page: www.cement.org/buildings.

Example 1

In this example, an interior column at the 1st floor level of a 7-story building is designed for the effects of gravity loads. Structural walls resist lateral loads, and the frame is nonsway.

Materials

• Concrete: normal weight (150 pcf), 3/4-in. maximum aggregate, f'_c = 4,000 psi

• Mild reinforcing steel: Grade 60 (f_y = 60,000 psi)

Loads

• Floor framing dead load = 90 psf

• Superimposed dead loads = 30 psf

• Live load = 100 psf (floor), 20 psf (roof)

Building Data

• Typical interior bay = 30 ft x 30 ft

• Story height = 12 ft-0 in.

The table below contains a summary of the axial loads due to gravity. The total factored load P_u is computed in accordance with Sect. 9.2.1, and includes an estimate for the weight of the column. Live load reduction is determined from ASCE 7-02. Moments due to gravity loads are negligible.

Floor	DL (psf)	LL (psf)	Red. LL (psf)	P_u (kips)	Cum. P_u (kips)
7	90	20	20.0	126.0	126.0
6	120	100	50.0	202.6	327.6
5	120	100	42.7	191.1	518.7
4	120	100	40.0	187.2	705.9
3	120	100	40.0	187.2	893.1
2	120	100	40.0	187.2	1,080
1	120	100	40.0	187.2	1,268

Use Fig. 5-1 to determine a preliminary size for the tied column at the 1st floor level.

Assuming a reinforcement ratio ρ_g = 0.020, for P_u = 1,268 kips, a 24" x 24" column is required.

Check if slenderness effects need to be considered.

Since the column is part of a nonsway frame, slenderness effects can be neglected when the unsupported column length is less than or equal to 12h, where h is the column dimension (ACI 318-05 Sect. 10.12.2).

12h = 12 x 24 = 288 in. = 24 ft > 12 ft story height, which is greater than the unsupported length of the column. Therefore, slenderness effects can be neglected.

To determine the required area of longitudinal reinforcement.

For a 24 x 24 in. column at the 1st floor level:

$A_s = 0.02 \times 24 \times 24 = 11.52 \text{ in.}^2$

Try 8-No. 11 bars ($A_s = 12.48 \text{ in.}^2$)

Check Eq. (10-2) of ACI 318-05:

$$\phi P_{n(max)} = 0.80\phi[0.85f'_c\,(A_g - A_{st}) + f_y\,A_{st}]$$
$$= 1{,}386 \text{ kips} > 1{,}268 \text{ kips O.K.}$$

3-No. 11 bars can be accommodated on the face of a 24-in. wide column with normal lap splices and No. 4 ties.

Determine required ties and spacing.

According to Sect. 7.10.5.1, No. 4 ties are required when No. 11 longitudinal bars are used.

According to Sect. 7.10.5.2, spacing of ties shall not exceed the least of:

16 long. bar diameters = 16 x 1.41
= 22.6 in. (governs)

48 tie bar diameters = 48 x 0.5
= 24 in.

Least column dimension = 24 in.

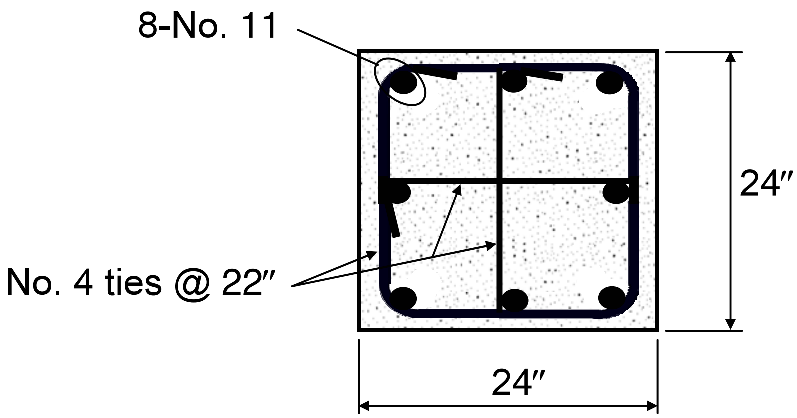
Check clear spacing of longitudinal bars:

$$\text{Clear space} = \frac{24 - 2\left(1.5 + 0.5 + \frac{1.41}{2}\right)}{2} - 1.41 = 7.9 \text{ in.}$$

Since the clear space between longitudinal bars > 6 in., cross-ties are required per ACI 318-05 Sect. 7.10.5.3.

Reinforcement details are shown below.

See ACI 318-05 Sect. 7.8 for additional special reinforcement details for columns.



Example 2

In this example, a simplified interaction diagram is constructed for an 18 in. x 18 in. tied column reinforced with 8-No. 9 Grade 60 bars (ρ_g = 8/18^2 = 0.0247). Concrete compressive strength = 4 ksi.

Use the simplified equations to determine the 5 points on the interaction diagram.

• Point 1: Pure compression

$$\phi P_{n(max)} = 0.80\phi A_g[0.85f'_c + \rho_g(f_y - 0.85f'_c)]$$

$$= 0.52 \times 18^2[(0.85 \times 4) + 0.0247\ (60 - (0.85 \times 4))]$$

$$= 808.3 \text{ kips}$$

• Point 2 (f_{st} = 0)

Layer 1:

$$1 - C_2\,\frac{d_1}{d_1} = 1 - 1(1) = 0$$

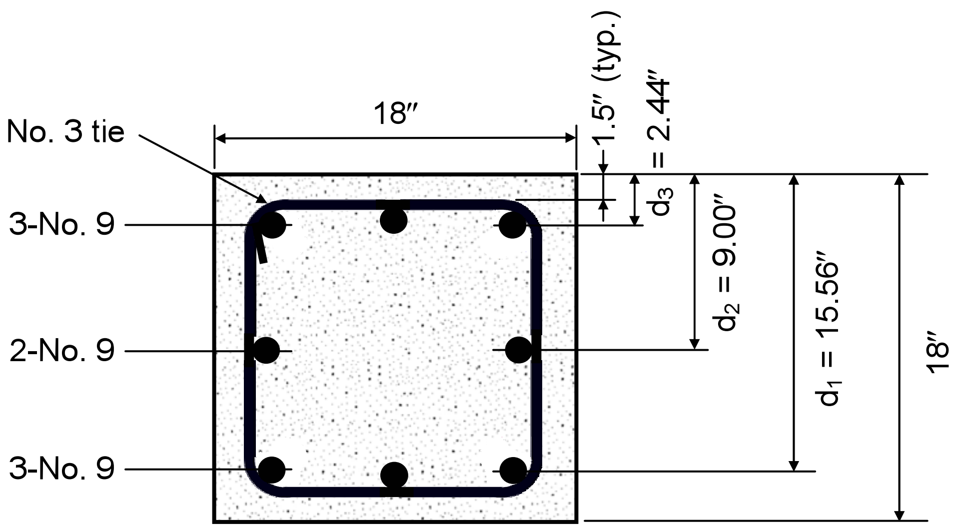
Layer 2:

$$1 - C_2\,\frac{d_2}{d_1} = 1 - 1\left(\frac{9.00}{15.56}\right) = 0.42$$

Layer 3:

$$1 - C_2\,\frac{d_3}{d_1} = 1 - 1\left(\frac{9.00}{15.56}\right) = 0.84$$

Since $1 - C_2\,(d_3/d_1) > 0.69$, the steel in layer 3 has yielded.



Therefore, set $1 - C_2\,(d_3/d_1) = 0.69$ to ensure that the stress in the bars in layer 3 is equal to 60 ksi.

$$\phi P_n = \phi\left[C_1 d_1 b + 87\sum_{i=1}^{n} A_{si}\left(1 - C_2\frac{d_i}{d_1}\right)\right]$$

= 0.65{(2.89 x 15.56 x 18)

+ 87[(3 x 0) + (2 x 0.42)

+ (3 x 0.69)]}

= 0.65 (809.4 + 253.2)

= 690.9 kips

$$\phi M_n = \phi\left[0.5C_1 d_1 b\left(h - C_3 d_1\right) + 87\sum_{i=1}^{n} A_{si}\left(1 - C_2\frac{d_i}{d_1}\right)\left(\frac{h}{2} - d_i\right)\right]/12$$

= 0.65{(0.5 x 2.89 x 15.56 x 18)

x (18-0.85 x 15.56)

+ 87[(3 x 0) (9 - 15.56)

+ (2 x 0.42) (9 - 9)

+ (3 x 0.69) (9 - 2.44)]} /12

= 0.65 (1,932.1 + 1,181.4) /12

= 168.6 ft - kips

Columns

• Point 3 (f_{st} = -0.5f_y)

Layer 1:

$$1 - C_2 \frac{d_1}{d_1} = 1 - 1.35\,(1) = -0.35$$

Layer 2:

$$1 - C_2 \frac{d_2}{d_1} = 1 - 1.35 \left(\frac{9.00}{15.56}\right) = 0.22$$

Layer 3:

$$1 - C_2 \frac{d_3}{d_1} = 1 - 1.35 \left(\frac{2.44}{15.56}\right) = 0.79, \text{ Use } 0.69$$

$$\phi P_n = \phi \left[C_1 d_1 b + 87 \sum_{i=1}^{n} A_{si}\left(1 - C_2 \frac{di}{d_1}\right)\right]$$

= 0.65{(2.14 x 15.56 x 18)

+ 87[(3 x -0.35) + (2 x 0.22)

+ (3 x 0.69)]}

= 0.65 (599.4 + 127.0)

= 474.9 ft - kips

$$\phi M_n = \phi \left[0.5 C_1 d_1 b\left(h - C_3 d_1\right) + 87 \sum_{i=1}^{n} A_{si}\left(1 - C_2 \frac{di}{d_1}\right)\left(\frac{h}{2} - d_i\right)\right] /12$$

= 0.65{(0.5 x 2.14 x 15.56 x 18)

(18 - 0.63 x 15.56)

+ 87[(3 x -0.35) (9 - 15.56)

+ (2 x 0.23) (9 - 9)

+ (3 x 0.69) (9 - 2.44)]} /12

= 0.65 (2,456.6 + 1,780.7) /12

= 229.1 ft - kips

Columns

• Point 4 ($f_{st} = -f_y$)

Layer 1:

$$1 - C_2 \frac{d_1}{d_1} = 1 - 1.69\ (1) = -0.69$$

Layer 2:

$$1 - C_2 \frac{d_2}{d_1} = 1 - 1.69 \left(\frac{9.00}{15.56}\right) = 0.02$$

Layer 3:

$$1 - C_2 \frac{d_3}{d_1} = 1 - 1.69 \left(\frac{2.44}{15.56}\right) = 0.74\text{, Use } 0.69$$

$$\phi P_n = \phi\left[C_1 d_1 b + 87\sum_{i=1}^{n} A_{si}\left(1 - C_2\frac{d_i}{d_1}\right)\right]$$

= 0.65{(1.70 x 15.56 x 18)

+ 87[(3 x -0.69) + (2 x 0.02)

+ (3 x 0.69)]}

= 0.65 (476.1 + 3.5)

= 314.0 kips

$$\phi M_n = \phi\left[0.5C_1 d_1 b\left(h - C_3 d_1\right) + 87\sum_{i=1}^{n} A_{si}\left(1 - C_2\frac{d_i}{d_1}\right)\left(\frac{h}{2} - d_i\right)\right]/12$$

= 0.65{[(0.5 x 1.70 x 15.56 x 18)

x (18 - 0.50 x 15.56)

+ 87[(3 x -0.69) (9 - 15.56)

+ (2 x 0.02) (9 - 9)

+ (3 x 0.69) (9 - 2.44)]} /12

= 0.65 (2,433.1 + 2,362.8) /12

= 260 ft - kips

• Point 5: Pure bending

Use iterative procedure to determine ϕM_n.

Try c = 4.0 in.

$\varepsilon_{s1} = 0.003 \left(\frac{c - d_1}{c}\right)$

$= 0.003 \left(\frac{4 - 15.56}{4}\right)$

$= -0.0087$

$f_{s1} = E_s \varepsilon_{s1}$

$= 29{,}000 \times (-0.0087) = -251.4$ ksi > -60 ksi, use $f_{s1} = -60$ ksi

$T_{s1} = A_{s1} f_{s1} = 3 \times (-60) = -180$ kips

$\varepsilon_{s2} = 0.003 \left(\frac{c - d_2}{c}\right)$

$= 0.003 \left(\frac{4 - 9}{4}\right)$

$= -0.0038$

$f_{s2} = E_s \varepsilon_{s2}$

$= 29{,}000 \times (-0.0038) = -108.8$ ksi > -60 ksi, use $f_{s2} = -60$ ksi

$T_{s2} = A_{s2} f_{s2} = 2 \times (-60) = -120$ kips

$\varepsilon_{s3} = 0.003 \left(\frac{c - d_3}{c}\right)$

$= 0.003 \left(\frac{4 - 2.44}{4}\right)$

$= 0.0012$

$f_{s3} = E_s \varepsilon_{s2}$

$= 29{,}000 \times (0.0012) = 33.9$ ksi

$C_{s3} = A_{s3} f_{s3} = 3 \times 33.9 = 102$ kips

$C_c = 0.85 f'_c ab$

$= 0.85 \times 4 \times (0.85 \times 4) \times 18$

$= 208$ kips

Total T = (-180) + (-120) = -300 kips

Total C = 102 + 208 = 310 kips

Since T ≈ C, use c = 4.0 in.

$$M_{ns1} = T_{s1}\left(\frac{h}{2} - d_i\right) = (-180)\left(\frac{18}{2} - 15.56\right)/12$$

$$= 98.4 \text{ ft - kips}$$

$$M_{ns2} = T_{s2}\left(\frac{h}{2} - d_2\right) = (-120)\left(\frac{18}{2} - 9\right)/12$$

$$= 0$$

$$M_{ns3} = C_{s3}\left(\frac{h}{2} - d_3\right) = 102\left(\frac{18}{2} - 2.44\right)/12$$

$$= 55.8 \text{ ft - kips}$$

$$M_n = 0.5C_c\,(h - a) + \sum_{i=1}^{3} M_{nsi} = \left[0.5 \times 208 \times (18 - 3.4)\right]/12 + 154.2$$

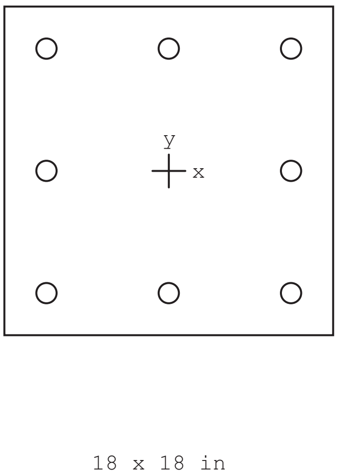
$$= 280.7 \text{ ft - kips}$$

$$\phi M_n = 0.9 \times 280.7 = 253 \text{ ft - kips}$$

Compare simplified interaction diagram to interaction diagram generated from the PCA computer program pcaColumn.

The comparison is shown on the next page. As can be seen from the figure, the comparison between the exact (black line) and simplified (red line) interaction diagrams is very good.

Columns



Code: ACI 318-05

Units: English

Run axis: About X-axis

Run option: Investigation

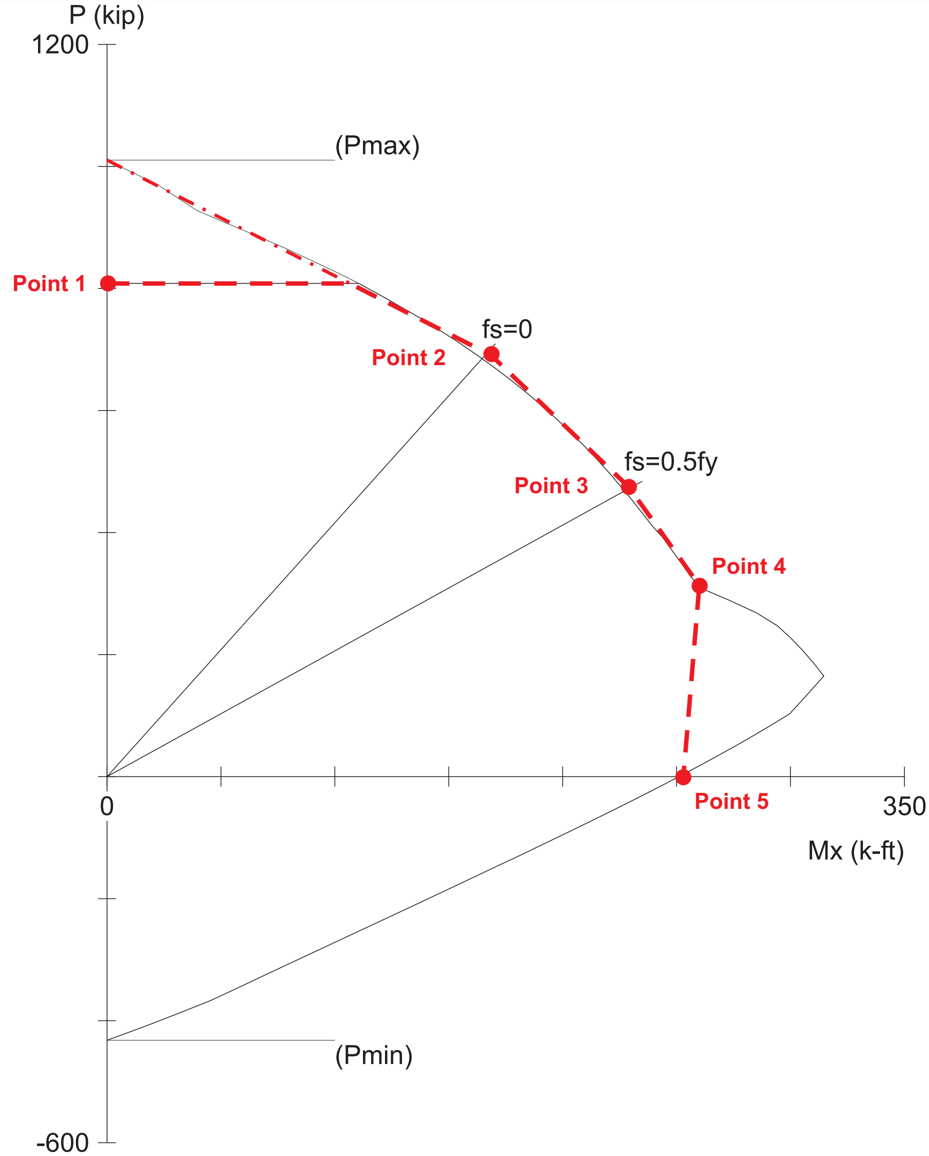
Slenderness: Not considered

Column type: Structural

Bars: ASTM A615

Date: 05/03/07

Time: 16:48:19



pcaColumn v4.00 Beta - May 3 2007. Licensed to: Portland Cement Association. License ID: 00000-0000000-4-2D2DE-2C8D0

File: C:\Data\Time Saving Design Aid\Column example.col

Project: Time Saving Design Aid-Columns

Column: Example 2		Engineer: DAA	
f'c = 4 ksi	fy = 60 ksi	Ag = 324 in^2	8 #9 bars
Ec = 3605 ksi	Es = 29000 ksi	As = 8.00 in^2	Rho = 2.47%
fc = 3.4 ksi	fc = 3.4 ksi	Xo = 0.00 in	Ix = 8748 in^4
e_u = 0.003 in/in		Yo = 0.00 in	Iy = 8748 in^4
Beta1 = 0.85		Clear spacing = 5.57 in	Clear cover = 1.74 in
Confinement: Tied	phi(a) = 0.8, phi(b) = 0.9, phi(c) = 0.65		

Columns

```
pcaColumn v4.00 Beta - May  3 2007 © Portland Cement Association              Page   2
Licensed to: Portland Cement Association. License ID: 00000-0000000-4-2D2DE-2C8D0  05/03/07
C:\Data\Time Saving Design Aid\Column example.col                             04:48 PM


  General Information:
  ====================
     File Name: C:\Data\Time Saving Design Aid\Column example.col
     Project:  Time Saving Design Aid-Columns
     Column:   Example 2                     Engineer: DAA
     Code:     Code: ACI 318-05              Units: English

     Run Option: Investigation               Slenderness: Not considered
     Run Axis:   X-axis                      Column Type: Structural

  Material Properties:
  ====================
     f'c   = 4 ksi                           fy   = 60 ksi
     Ec    = 3605 ksi                        Es   = 29000 ksi
     Ultimate strain = 0.003 in/in
     Beta1 = 0.85

  Section:
  ========
     Rectangular: Width = 18 in              Depth = 18 in

     Gross section area, Ag =  324 in^2
     Ix =  8748 in^4                         Iy =  8748 in^4
     Xo =  0 in                              Yo =  0 in

  Reinforcement:
  ==============
     Rebar Database: ASTM A615
     Size Diam (in) Area (in^2)   Size Diam (in) Area (in^2)   Size Diam (in) Area (in^2)
     ---- --------- -----------   ---- --------- -----------   ---- --------- -----------
     #  3      0.38        0.11   #  4      0.50        0.20   #  5      0.63        0.31
     #  6      0.75        0.44   #  7      0.88        0.60   #  8      1.00        0.79
     #  9      1.13        1.00   # 10      1.27        1.27   # 11      1.41        1.56
     # 14      1.69        2.25   # 18      2.26        4.00

     Confinement: Tied; #3 ties with #10 bars,  #4 with larger bars.
     phi(a) = 0.8,  phi(b) = 0.9,  phi(c) = 0.65

     Layout: Rectangular
     Pattern: All Sides Equal  (Cover to longitudinal reinforcement)
     Total steel area, As = 8.00 in^2 at 2.47%
     8 #9   Cover = 1.735 in

  Control Points:
  ===============
                          Axial Load P     X-Moment     Y-Moment  N.A. depth      Phi
     Bending about                 kip         k-ft         k-ft          in
     --------------------- ------------ ------------ ------------ ----------- --------
      X @ Pure compression       1010.4           -0           -0       50.59    0.650
        @ Max compression         808.3          110            0       18.49    0.650
        @ fs = 0.0                685.1          165            0       15.70    0.650
        @ fs = 0.5*fy             468.1          227            0       11.68    0.650
        @ Balanced point          311.1          259            0        9.29    0.650
        @ Tension Control         164.9          315            0        5.89    0.900
        @ Pure bending              0.0          251            0        3.90    0.900
        @ Pure tension           -432.0            0           -0        0.00    0.900

     -X @ Pure compression       1010.4           -0           -0       50.59    0.650
        @ Max compression         808.3         -110           -0       18.49    0.650
        @ fs = 0.0                685.1         -165           -0       15.70    0.650
        @ fs = 0.5*fy             468.1         -227            0       11.68    0.650
        @ Balanced point          311.1         -259           -0        9.29    0.650
        @ Tension Control         164.9         -315           -0        5.89    0.900
        @ Pure bending              0.0         -251           -0        3.90    0.900
        @ Pure tension           -432.0            0           -0        0.00    0.900


     *** Program completed as requested! ***
```